

How Does the Sun Shine?

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Abstract

Assuming that MSW neutrino oscillations occur and ignoring all solar physics except for the constraint that nuclear fusion produces the solar luminosity, we show that new solar neutrino experiments are required to rule out empirically the hypothesis that the sun shines via the CNO cycle.

In 1939, Bethe [1] showed that the energy required to enable the sun to shine for several billion years could be obtained by two alternative sets of reactions, which have come to be known as the pp chain and the CNO cycle. Although Bethe's original calculations favored the CNO reactions for the solar energy source, detailed solar models developed in the late 1950s and early 1960s indicated that the pp chain was dominant in the sun. Many authors (e.g., [2–5]) have summarized the specific reactions that occur, according to current understanding, in the pp chain and the CNO cycle.

Do solar neutrino experiments confirm the theoretical calculations that indicate that the sun shines primarily by the pp fusion chain? The predominant opinion (to which we also subscribed before doing the calculations described in this paper) seems to be [6] that the pp neutrinos have been observed in the gallium solar neutrino experiments, GALLEX [7] and SAGE [8], establishing experimentally the predominance of the pp chain. The reasons for this view include the approximate agreement between the total observed rate in the gallium experiments, 74 ± 8 SNU and the total rate, 73 SNU, predicted in the standard solar model (SSM) [9] to arise from only the pp (and pep) neutrinos. The observed rate is about half the predicted standard rate from all neutrino sources. In addition, the CNO

cycle contributes only about 2 % to the total solar luminosity in the standard solar model, with the overwhelming contribution (98%) coming from the pp chain. Perhaps even more suggestive is the fact that the rare, high-energy ^8B neutrinos, produced in the pp chain, have been observed in the Kamiokande experiment [10].

However, there is no direct experimental evidence that pp neutrinos have been detected. Only the gallium experiments have a sufficiently low energy threshold to observe pp neutrinos and these radiochemical experiments do not have any way of recording the energies of the neutrinos that produce ^{71}Ge from ^{71}Ga . The ^{71}Ge detected in the gallium experiments could, in principle, be produced by low energy pp neutrinos, by somewhat higher energy CNO neutrinos, or by a linear combination of neutrino fluxes from the various nuclear reactions that are believed to create neutrinos in the solar interior.

The combined predictions of standard electroweak theory and standard solar models provide unique and easily testable consequences, which the existing solar neutrino experiments suggest may be not correct. Once one admits the possibility of new physics altering the solar neutrino spectrum, it becomes much more difficult to make unique inferences from the neutrino experiments.

We show in this paper that, if neutrino oscillations can occur, the four operating solar neutrino experiments (chlorine [11], Kamiokande, GALLEX, and SAGE) are consistent with a hypothetical solar neutrino spectrum in which CNO reactions produce essentially all of the solar luminosity. In fact, there is a one-parameter family of such “solutions” to the solar neutrino problems. These solutions are inconsistent with the standard solar model, but they are consistent with the luminosity constraint, i. e., the fusion energy release to the star associated with the neutrino production equals the observed solar luminosity. In addition, we require that our solutions satisfy the inequality [12] between neutrino fluxes, ϕ , that follows from the set of nuclear reactions that produce ^7Be and ^8B neutrinos: $\phi(^7\text{Be}) + \phi(^8\text{B}) \leq \phi(\text{pp}) + \phi(\text{pep})$. This inequality expresses the fact that ^7Be and ^8B neutrinos are produced by electron capture on ^7Be , that ^7Be itself is produced by the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction, and that a pp or pep reaction is required to produce each ^3He nucleus. We do not require that

the ratio of $\phi(^7\text{Be})$ to $\phi(^8\text{B})$, or the ratio of the reaction rates for $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ and $^3\text{He}(^4\text{He}, \gamma)^7\text{Be}$, be equal to the values computed in a standard solar model.

Before describing the solutions, we want to make clear that we do not believe that the sun shines by the CNO cycle. The successes of the standard solar model are too great for us to believe that a radically different solar model could explain all the observations, especially the many thousands of precisely measured helioseismological frequencies that are well described by the standard solar model [13,14]. The purpose of our work is to illustrate the limits of what can be learned from radiochemical solar neutrino experiments, which do not measure the energies of individual events, and to emphasize the importance of future experiments with energy resolution.

Table I describes the CNO analogue of the by-now “conventional” small-mixing angle MSW solution [15]. The conventional neutrino oscillation solutions presume that the neutrino spectrum created in the interior of the sun is similar to what is predicted by the standard solar model, i.e., most solar neutrinos are produced by the pp reaction. The solutions presented here are radically different from what would be implied by a standard solar model. The second column of Table I gives the ratio of the total flux from each neutrino source to the maximum flux from that source permitted by the luminosity constraint¹. In the specific solution described in the table, the pp neutrinos represent 0.05% of the total solar luminosity; the CNO neutrinos [$\phi(^{13}\text{N}) = \phi(^{15}\text{O}) = \phi(\text{CNO})$] correspond to 99.95% of the total energy output. The total ^8B neutrino flux (all flavors) is about 1.5 times the standard solar model flux [9]. The third, fourth, and fifth columns give the fractional contribution of each neutrino source to the total observed rate in each of the operating experiments. The last two rows of Table I show that the observed and the calculated event rates are in

¹ The maximum fluxes allowed by the luminosity constraint and the nuclear physics inequality, $\phi(^7\text{Be}) + \phi(^8\text{B}) \leq \phi(\text{pp}) + \phi(\text{pep})$, are [12]: $6.51 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$ (pp); $7.16 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$ (pep); $3.33 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$ (^7Be); $4.32 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$ (^8B); and $3.41 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$ (CNO).

excellent agreement.

The CNO solutions were found by a computer search that considered (over a numerical grid) all relevant values of the neutrino fluxes, and mixing parameters, that are consistent with the luminosity constraint. After choosing a (large) CNO flux, the ^8B flux was selected to lie within the range that can be consistent with the Kamiokande experiment (taking account of the quoted experimental errors and the possibility that neutrino oscillations may occur). Since the adopted CNO flux is large, the luminosity constraint bounds the pp and ^7Be fluxes to such small values that they do not contribute significantly to the event rates in any of the experiments. There is therefore a one-parameter family (an infinite set) of CNO solutions in which the small residual luminosity is divided between pp and ^7Be neutrinos. For the explicit solution given in Table I, we (arbitrarily) maximized the ^7Be contribution to the luminosity not associated with CNO and ^8B neutrinos. For each chosen set of neutrino fluxes, standard techniques were used to compute the survival probabilities [16] for electron-type neutrinos and then to compare the calculated event rates (see [17]) in the four operating experiments with the observed rates.

Figure 1 shows the computed survival probability as a function of energy for the CNO solution given in Table I. The survival probability is defined as the probability that an electron-type neutrino created in the sun will be detected as an electron-type neutrino when it reaches the earth. Because different neutrino sources are produced at somewhat different positions in the solar interior, the computed survival probability at a given neutrino energy depends slightly upon which neutrino source one is considering. The specific curve shown in Figure 1 was computed for ^8B neutrinos. For our purposes, it is a good approximation to consider the illustrated survival probability as generic.

The survival probability is small in the region (1 MeV to 10 MeV) that is most important for the ^8B neutrinos; it is approximately $\exp(-17 \text{ MeV}/\text{energy})$ for neutrinos with energies above 4 MeV. For the 0.86 MeV ^7Be line, the survival probability is 0.11. The survival probability rises steeply at energies below the threshold (0.8 MeV) of the Homestake detector. The average energies of the ^{13}N and the ^{15}O neutrinos are, respectively, 0.7 MeV

and 1.0 MeV and the end point energies are 1.2 MeV and 1.7 MeV. Therefore, the CNO contribution to the chlorine detector is strongly suppressed (by about a factor of 34), while the CNO contribution to the gallium experiments is somewhat less suppressed (by a factor of about 9). In this way, the low energy neutrinos from the CNO cycle can produce a signal in the gallium detectors that is comparable to the predicted pp signal in the standard model calculations, without producing an excessive contribution in the chlorine detector.

Figure 2 shows the neutrino parameters for the MSW solutions that correspond to at least 99.95% of the solar luminosity being produced by CNO neutrinos. The best-fit CNO solution is indicated by a dark circle in Figure 2 and the conventional (pp-dominated) solutions [16,17] are indicated by dark triangles. Unlike the familiar MSW plots in which the standard solar model is assumed to be valid (within estimated uncertainties), the ^8B flux is treated as a free parameter in the calculations that give the results shown in Figure 2. For specificity, we required $\chi^2 \leq 5.99 + \chi_{\min}^2$ for the fits to the operating experiments used in drawing the contours in Figure 2; this requirement corresponds to a 95% confidence level for the two neutrino mixing parameters (Δm^2 and $\sin^2(2\theta)$). The minimum value of χ^2 for the best-fitting parameters is $\chi_{\min}^2 = 0.28$ (CNO solution), which can be compared with [12] $\chi_{\min}^2 = 0.31$ (small mixing angle pp-based solution) and $\chi_{\min}^2 = 2.5$ (large mixing angle pp-based solution).

We concentrate in this paper on the most extreme cases in which essentially all the solar luminosity is generated by the CNO cycle. The computer search did find, of course, other sets of solutions in which the CNO contribution can range anywhere from 0% to almost 100%.

The lack of an observed day-night effect in the Kamiokande experiment [10] rules out a large mixing-angle “essentially all” CNO solution. The average rates (ignoring day-night differences) in the four operating solar experiments are consistent with a CNO solution and neutrino parameters $\Delta m^2 = 7 \times 10^{-6} \text{ eV}^2$ and $\sin^2 2\theta = 0.14$. This ruled-out CNO solution is the analogue of the conventional large mixing angle (pp-dominated) MSW solution shown in Figure 2, but the unacceptable CNO solution has a smaller Δm^2 and mixing angle than

the conventional large mixing angle solution.

The survival probabilities depend somewhat on the calculated density profile and the neutrino production regions that are derived from a solar model, and therefore are slightly model dependent. The allowed regions were computed using survival probabilities calculated for three different solar models: 1995 [9], 1992 [18], and 1988 [13]. Figure 2 shows that the final numerical results are not sensitive to which reference solar model is used.

The critical reader may object that we have not presented a self-consistent solution for which the neutrino survival probabilities are computed from a detailed solar model in which the energy production is dominated by CNO reactions. This objection is valid. Our goal in writing this paper is to show that solar neutrino experiments with the ability to measure the energies of individual low-energy events are required in order to establish empirically that the sun shines by the pp, not the CNO, reactions. We are not trying to present a self-consistent CNO-based solar model. When all the correct physics is included in a detailed solar model, the theoretical calculations show that the energy production is dominated by pp not CNO reactions. However, helioseismological measurements indicate [14] that the solar sound velocity (closely related to the density profile) does not differ significantly (less than or of order a percent) from the standard model profile, as far as the helioseismological measurements have probed (down to about 10% of the solar radius). So, if there were a self-consistent CNO model that agreed with the helioseismological measurements, then it would have a density profile similar to the standard models used here.

CNO solutions can be found with a wide range for the ratio of electron capture to proton capture on ${}^7\text{Be}$, which determines the ratio, R , of ${}^7\text{Be}$ to ${}^8\text{B}$ neutrino fluxes. The specific solution given in Table I has $R \simeq 1$, but we have found solutions with R values varying from 0 to 10^2 (in the latter case 98% of the solar luminosity is in the form of CNO neutrinos). Larger values of R can be found if one is willing to consider solutions in which the fraction, $f(\text{pp})$, of the solar luminosity that derives from pp reactions exceeds 2% ($R_{\text{max}} \propto f(\text{pp})$).

Assuming vacuum neutrino oscillations can occur (but not MSW oscillations), we have not been able to find solutions, consistent with the luminosity constraint, in which the CNO

energy generated dominated the solar energy production. The largest CNO contribution we found was 12% of the solar luminosity; this value corresponds to vacuum oscillation parameters of $\Delta m^2 = 6.4 \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta = 1.0$. The vacuum oscillation solutions have characteristically oscillatory behavior as a function of neutrino energy, which makes it difficult to suppress electron-type neutrinos over a very large range of energies (cf. Figure 1).

For solar neutrino experiments under development, Table II gives in the second column the event rates predicted by the “essentially-all” MSW-CNO solution. Assuming for comparison the correctness of the standard solar model (in which nearly all the energy is produced by the pp chain), columns three, four, and five, give the predicted rates [12] for the conventional MSW and vacuum neutrino oscillation solutions. The error bars in Table II were computed [12] by allowing Δm^2 and $\sin^2 2\theta$ to vary over the range that is consistent, at 95% confidence level, with the four operating solar neutrino experiments. For the CNO solution, the total ^8B neutrino flux varies between 0.7 and 2.4 times the standard model value.

Table II shows that measurements of the ^8B neutrino rates by SuperKamiokande [19], SNO [20], and ICARUS [21] will not be able to rule out the essentially-all CNO MSW solution. Nevertheless, these experiments are expected to be able to demonstrate definitively if physics beyond the simplest version of standard electroweak theory is required to describe solar neutrino experiments and, if new physics is required, to make relatively accurate determinations of some neutrino mixing parameters with only modest guidance from theoretical models.

Three future experiments, BOREXINO [22], HELLAZ [23], and HERON [24], have been proposed that would measure directly the fluxes of low energy solar neutrinos. BOREXINO is designed to take advantage of the characteristic ‘box’ shape [4] of the recoil electron energy spectrum from a neutrino line, ^7Be . The CNO solution predicts that ^7Be neutrinos will be unobservable in BOREXINO, with an interaction rate of 10^{-4} the standard model prediction. HELLAZ and HERON are intended to measure the fundamental pp neutrinos, which the CNO solution predicts to be unobservably rare, $\sim 3 \times 10^{-4}$ the standard model

prediction. BOREXINO, HELLAZ, and HERON should all observe, according to the CNO solution, a low-energy continuous spectrum dominated by neutrinos from ^{13}N and ^{15}O decay. The CNO contribution to the event rate would be about three times the event rate predicted by the standard model in the relevant energy ranges (300 keV to 665 keV for ^7Be neutrinos in BOREXINO, 100 keV to 260 keV for pp neutrinos in HELLAZ/HERON).

In conclusion, we want to stress again that we have ignored in this paper all considerations based upon either theoretical solar models or helioseismology. We have focused instead on what can be inferred empirically from solar neutrino experiments if neutrino oscillations occur. If neutrino oscillations occur, then experiments with powerful diagnostic capabilities (high counting rates, good energy and time resolution for individual events) are required in order to determine empirically the solar neutrino spectrum.

ACKNOWLEDGMENTS

J.N.B. acknowledges support from NSF grant #PHY92-45317. M. F. acknowledges generous support from the Fuji Xerox Corporation. The work of P.I.K. was partially supported by funds from the Institute for Advanced Study. For valuable comments on an original version of this manuscript, we are grateful to I. Dostrovsky, F. Dyson, R. Eisenstein, W. Hampel, E. Lisi, S. Parke, P. Parker, R. Raghavan, H. Robertson, M. Spiro, J. Wenesser, J. Wilkerson, and L. Wolfenstein.

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TABLES

TABLE I. For the “essentially-all CNO solution,” the individual contributions to the calculated event rates are given for the operating solar neutrino experiments. The second column of the table gives the ratio, $\text{Flux}/(\text{Flux})_{\text{max}}$, of the total flux from each neutrino source to the maximum flux [12] from that source permitted by the luminosity constraint. For example, the pp flux is only 0.03% of the maximum pp flux allowed by the luminosity constraint; the ^{13}N and ^{15}O fluxes are approximately equal to their maximum allowed values. The survival probabilities for electron-type neutrinos are computed for the MSW small-angle solution with $\Delta m^2 = 8 \times 10^{-6} \text{ eV}^2$ and $\sin^2 2\theta = 9 \times 10^{-3}$.

Neutrino Source	Fluxes $\text{Flux}/(\text{Flux})_{\text{max}}$	Cl (SNU)	Ga (SNU)	Kamiokande (Observed/SSM)
pp	3×10^{-4}	—	0.0	—
pep	6×10^{-7}	0.0	0.0	—
^7Be	3×10^{-4}	0.0	0.0	—
^8B	2×10^{-4}	1.71	3.4	0.44
^{13}N	1.00	0.69	49.6	—
^{15}O	1.00	0.16	19.4	—
Total		2.56	72.4	0.44
Observed		2.55 ± 0.25	74.0 ± 8.0	0.44 ± 0.06

TABLE II. The ratios of the event rates predicted by the CNO solution to the rates given by the combined standard (solar and electroweak) model are presented in column two for solar neutrino experiments under development. The corresponding event rates predicted by the standard solar model (nearly all pp energy production) and the small mixing angle MSW (SMA), large mixing angle MSW (LMA), and vacuum (Vac) neutrino oscillation solutions are shown for comparison in columns three, four and five, respectively. The last row of the table refers to the double ratio [12] of neutral current to charge current event rates in the SNO experiment, i. e., the neutral to charged current event ratio calculated assuming neutrino oscillations divided by the ratio predicted by the standard model.

Experiment	CNO	SSM	SSM	SSM
	(SMA)	(SMA)	(LMA)	(Vac)
SuperKamiokande	$0.40^{+0.14}_{-0.13}$	$0.41^{+0.19}_{-0.13}$	$0.34^{+0.09}_{-0.06}$	$0.31^{+0.25}_{-0.06}$
SNO	$0.22^{+0.09}_{-0.09}$	$0.32^{+0.23}_{-0.16}$	$0.22^{+0.23}_{-0.06}$	$0.19^{+0.23}_{-0.10}$
ICARUS	$0.24^{+0.09}_{-0.09}$	$0.34^{+0.23}_{-0.18}$	$0.22^{+0.11}_{-0.06}$	$0.23^{+0.20}_{-0.12}$
(NC/CC) _{DR}	$6.8^{+11.5}_{-5.0}$	$3.1^{+1.8}_{-1.3}$	$4.4^{+2.0}_{-1.4}$	$5.2^{+5.8}_{-2.9}$

FIGURES

FIG. 1. Survival Probabilities. The probability for an electron-type neutrino created in the sun to be detected as an electron-type neutrino when it reaches a terrestrial detector is given as a function of energy for the CNO solution presented in Table I. The numerical results shown in the figure were obtained for ^8B neutrinos using the 1995 standard solar model [9]. Similar results were obtained with other neutrino sources (produced in the model with somewhat different probabilities at different solar radii) and with the 1988 and 1992 solar models.

FIG. 2. CNO-MSW Solutions. The allowed regions at 95% C.L. for $\sin^2 2\theta$ and Δm^2 are shown for the CNO-MSW solutions of the solar neutrino problems. The enclosed regions comprise the values of the neutrino mixing parameters for which a statistically acceptable solution exists and for which at least 99.95% of the solar energy generation arises from CNO fusion reactions. The dotted and dash-dotted line contours were computed using the solar models [13] (1988) and [18] (1992); the full line contour is for the most recent solar model [9] (1995). The CNO-dominated MSW solution presented in Table I is indicated by a dark circle. The conventional [12,17] pp-dominated MSW solutions are marked by filled-in triangles.

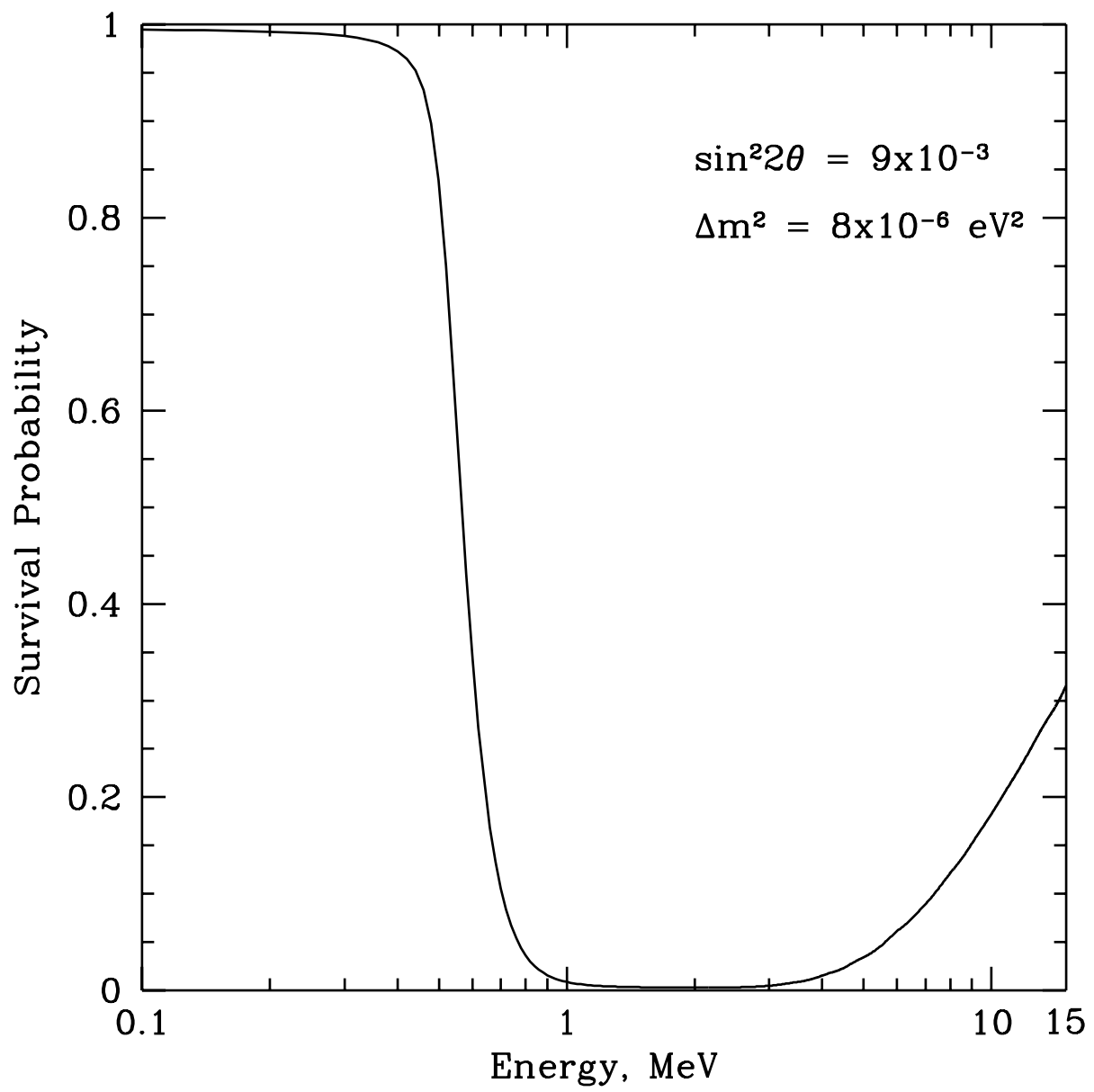


Fig.1

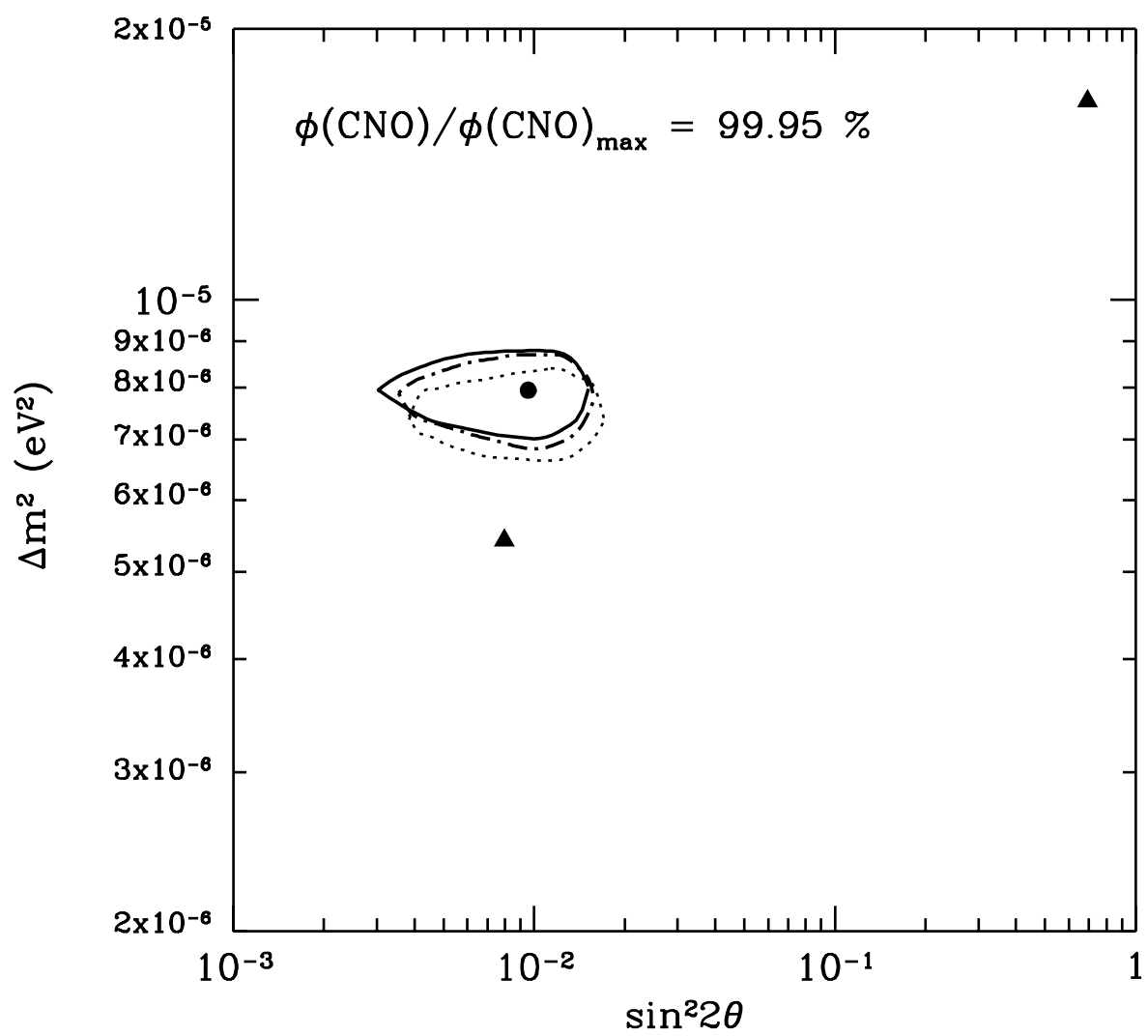


Fig.2